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MESS

(Misalignment Estimation Software System) for In-Flight Alignment and Calibration of Spacecraft Attitude Sensors

by

R. desJardins
Goddard Space Flight Center
Greenbelt, Maryland

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ABSTRACT

Spacecraft with very high precision slewing and attitude maintenance requirements typically combine a number of high precision celestial reference sensors with one or more inertial reference sensors. It then becomes necessary to have the capabilities of aligning these various attitude reference sensors relative to one another in flight, and of making in-flight calibrations of the slew angle scale factors and drift rates of the gyros. The requirement to do this on the OAO led to the system of ground and spacecraft procedures and analysis known as MESS. Attitude sensor errors arising from preplanned spacecraft activity are returned to the OAO support computer data base. Then the MESS subroutine system correlates these errors with the spacecraft activity and processes these data to arrive at estimates of the sensor alignment and calibration parameter values which caused the errors. MESS then automatically computes correction factors for the computer model of the spacecraft, so that subsequent spacecraft commands will reflect compensations for the misalignments and calibration deviations detected.

1. Introduction

MESS (Misalignment Estimation Software System) is a system of subroutines, data sets and spacecraft and ground systems operational procedures for estimating the inflight values of certain attitude sensor alignment and calibration parameters on orbiting spacecraft. Typically MESS is applicable to spacecraft with very high precision slewing and attitude maintenance requirements. Such spacecraft will normally combine a number of high precision external reference sensors (star trackers, sun sensors, possibly earth sensors) with one or more internal reference sensors (inertial reference assemblies). To achieve very high precision in slewing and pointing, it becomes essential to be able to align these various attitude reference sensors relative to one another in flight, and also to calibrate in flight the slew angle scale factors and drift rates of the gyros.

The version of MESS currently implemented, MESS/OAO, was developed for the Orbiting Astronomical Observatory B (OAO-B). MESS/OAO has been used off-line for some alignment tasks on OAO-A2 and a modified version is being developed for OAO-C. OAO-B contained five gimballed star trackers (GST), a boresighted star tracker (BST), a fine error sensor (FES) utilizing the experiment optics, and an extemely stable inertial reference unit (IRU). However, the MESS architecture and mathematical and operations analysis are quite general, in the sense that a version of MESS can easily be implemented to accommodate an arbitrary three-axis stabilized spacecraft, and arbitrary attitude sensors aboard that spacecraft.

For example, MESS/OAO is capable of estimating the inflight values of the following parameters:

- The misalignments and zenith angles of the gimballed star trackers (GST).
- The misalignments of the boresighted star tracker (BST) and the GEP fine error sensor (FES).
- The slew axis misalignments and slew angle scale factors of the inertial reference unit (IRU).
- The drift rates of the IRU gyros.

MESS estimates are all made in conceptually the same way, as follows. If all the parameters in question had their nominal values, then under equilibrium conditions, there would be no errors in the various attitude sensors. That is, each GST would track its star with zero errors, the IRU would slew the spacecraft to its target attitude with zero errors, etc. In point of fact, however, errors do occur in these sensors. Some of these errors are random in nature, but systematic errors (biases) are also present in general. For most of these systematic errors, the error-causing mechanisms have been identified and modeled as first-order corrections to various spacecraft characteristic parameters.

The MESS system reads the various attitude sensor errors from the telemetry returning from the spacecraft, correlates the errors with the spacecraft activity which produced them, and then processes these correlated data to arrive at best corrected estimates of the parameter values which would cause such errors. These estimated parameter values subsequently become the new nominal values, putatively producing zero (or at least significantly smaller) errors.

2. MESS Exercises

Each MESS capability for aligning or calibrating an attitude sensor corresponds to a well-defined segment of spacecraft activity, called a MESS Exercise. The activity specified for each MESS Exercise is designed to produce the type and number of attitude sensor errors which will yield mathematically optimal estimates of the underlying sensor alignment and calibration parameters.

The sequence of ground system activity which occurs before and after a MESS Exercise is shown schematically in Figure 1. This activity begins with an analysis by the MESS Cadre of the alignment requirements for a particular sensor. Using the mathematical model of the spacecraft from the data base, a MESS Exercise is generated appropriate for the alignment required. The commands which will effect the Exercise are then fabricated, and uplinked to the spacecraft at the appropriate time.

After the spacecraft performs each segment of the Exercise, the resulting attitude sensor errors are downlinked to a storage disk on the MESS computer. When the total Exercise has been completed, the MESS Program processes the data stored on the disk to arrive at estimates of the alignment and calibration parameter values which they imply. When these values have been validated by a MESS Analyst, they are entered into the data base to revise the spacecraft model. Subsequent commands fabricated against this revised model should then result in improved spacecraft slewing and attitude control.

Four basic Exercises are used in MESS/OAO. These Exercises are performed in a particular order during the early mission orbits, so that the results

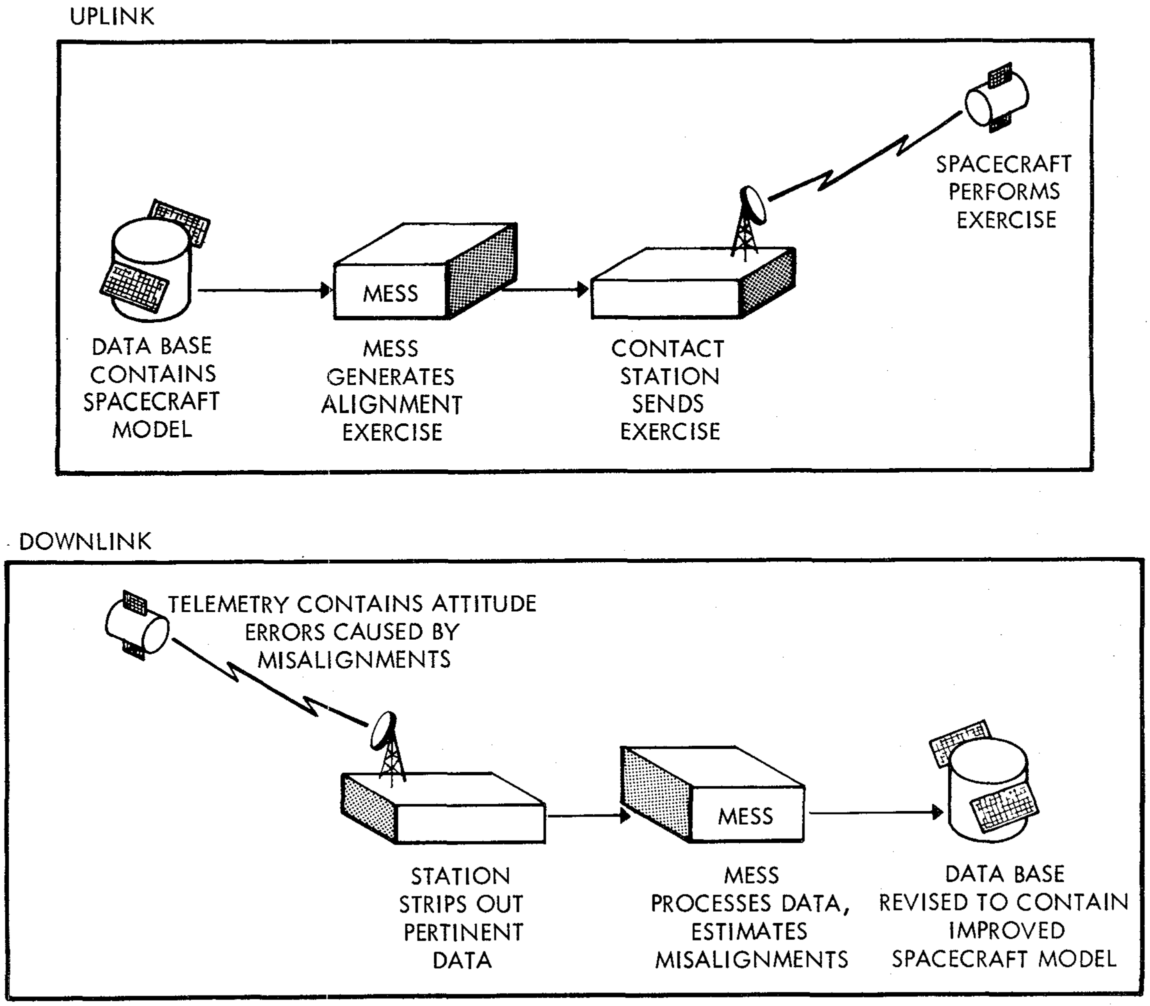


Figure 1. Sequence of Events for MESS Exercise

obtained by MESS from any given Exercise can be reflected in the commands sent to the spacecraft to effect the next Exercise.

The DRF Exercise for estimating the drift rates of the IRU gyros results in a nominally drift-free gyro platform. The GST Exercise for aligning the gimballed star trackers relative to one another results in an internally self-consistent set of gimballed stellar reference sensors. Then the PAX Exercise is performed to align the pointing axis sensors relative to the consistent GST set. Finally, when all stellar sensors have been aligned into a consistent set, the IRU Exercise is performed to determine the alignment of each IRU slewing axis relative to the stellar reference, and to calibrate the slew angle scale factors for precise slew angle sensing.

2.1. The DRF Exercise. IRU gyro drift causes gradual divergence between the celestial reference maintained by the stars in fixed space and the inertial platform reference defined by the gyro errors. If it were possible for the spacecraft to hold with a combination of stellar sensors on a fixed star pattern for a long interval of time (several orbits), the gyro drift would appear as a gradual buildup in the gyro error registers. Conversely, if the spacecraft were to hold on IRU for a similar time interval, the control system would cause the spacecraft to follow the inertial platform reference as it drifted and hence the stellar sensors would display a gradual buildup of errors. Of these two possible ways of observing gyro drift, the first is computationally more attractive, since the error readouts from each channel give the gyro error buildup directly.

Unfortunately, however, the spacecraft is unable to hold to a fixed stellar reference throughout the orbit. For when the stellar pattern changes as a

guide star becomes occulted or unocculted, the spacecraft attitude will change slightly due to misalignments of the trackers. Thus IRU error readings corresponding to a second pattern cannot be combined with readings corresponding to the first pattern.

The star patterns do tend to repeat, however, once per orbit. When a pattern repeats at two separated time points, the spacecraft can be put under stellar control at each, and the IRU errors noted. The change in error for each channel, divided by the time interval between readings, gives the basic error buildup rate for that channel. If such readings are available at several separated time points for the same pattern, good statistical (least-squares) estimates of the rates for each channel can be obtained.

Furthermore, stellar patterns other than the initial pattern can be used to provide additional independent estimates of the basic error rates. These data are processed in precisely the same way as data from the initial pattern. In fact, so long as data taken from any one pattern at one time is combined only with data from that same pattern taken at other times, the error readings will not be contaminated with spurious IRU platform movement due to pattern changes. Thus each pattern which repeats periodically can be used to obtain independent estimates of the basic gyro drift rates.

Against the preceding background discussion, the DRF Exercise philosophy can now be presented. An interval of about three orbits is set aside during which several different stellar patterns will be available and will repeat cyclically during successive station passes. The IRU error registers are reset to zero at the beginning of the Exercise, and then not reset again during the Exercise.

Thus the IRU reference is allowed to drift. Each time the spacecraft is available over a station, it is placed under stellar control at whatever pattern is available and allowed to settle out. A telemetry transmission containing attitude sensor data is then sent to the MESS/OAO computer. The data of interest in this transmission are the IRU errors, the time point at which the data are taken, and the GST configuration prevailing (gimbal angles and tracker status bits). Such transmissions are simply stored on a disk until the Exercise is completed.

When the last transmission has been received, MESS DRF processing is initiated. The telemetry received is merged with the known spacecraft attitude and characteristics and passed to the DRF components of MESS. These components automatically determine the stellar pattern which prevailed during each transmission, and then sort the transmissions into groups, each group corresponding to one pattern. When this has been done, the least-squares error rate estimates for each channel, for each pattern independently, are derived and printed out, and MESS DRF processing halts. The print out, containing the several independent estimates of drift rate in each channel, is then handed over to S & C (Stabilization and Control) personnel for interpretation. These personnel determine the numerical drift rate value to adopt for each channel and the factors which must be applied to the gyros to compensate for these adopted drift rate values. These compensation factors may then be commanded into the spacecraft from the ground during a subsequent contact.

2.2. The GST Exercise. The gimballed star trackers on OAO in an ideal sense constitute angle-measuring devices. If they made their measurements perfectly, it would be possible to determine, by mathematical analysis of the

gimbal angles, the angle between two given stars being tracked by a given pair of star trackers. Due to unknown misalignments of the trackers, however, the commanded gimbal angles drive the trackers to a configuration which does not exactly match the true angle between their guide stars, and hence when allowed to track these stars, the star trackers take on errors caused by the misalignments. The mathematical analyst can derive the relationship between the misalignments and the corresponding errors. Different stellar configurations will provide different equations relating the misalignments to the errors. Given a sufficient number of sufficiently different such equations, and given the errors, it is possible to solve for the misalignments which produced them. The purpose of the GST Exercise is to provide a number of different stellar configurations so that the GST misalignments can be determined.

The GST Exercise is performed as follows. A time interval of at least one orbit is set aside during which the spacecraft will make passes over a number of different stations, so that different stellar configurations will be available. At the beginning of each pass, the spacecraft is put under stellar control at any convenient stellar pattern, the IRU is reset, and then the spacecraft is placed under IRU control for the remainder of the pass. In general, several of the trackers will each now have several guide stars available within their gimbal capability. All such trackers are then gimballed together, each to such an available star. When the trackers have all acquired their stars and settled out, a burst of attitude telemetry data are taken and sent to the MESS computer. The data of interest in this transmission are the GST configuration and errors. Since the spacecraft is being held on IRU, these errors can be averaged together to remove the measurement jitter. The trackers are then all gimballed to

different stars and another transmission sent to the MESS computer. Several different such configurations can be viewed during each pass, for a number of passes at different stations, and a transmission sent to the MESS computer for each. Such transmissions are simply stored in a data set until the Exercise is completed.

When the last transmission has been received, MESS GST processing is initiated. The telemetry received is merged with the spacecraft attitude and characteristics and passed to the GST components of MESS. These components compute, for each pair of stars in each configuration, the angle between the stars as measured by the misaligned trackers. Coefficients are then computed for the linear equation which relates the unknown misalignments of the two trackers in each possible pair to the deviation of the measured angle from the true angle between that pair of guide stars. Such equations are collected into a linear system of equations in the unknown misalignments, and this system is solved in the least-squares sense. The resulting estimated misalignments are relative misalignments, in the sense that they are nominally consistent among the tracker themselves, but still contain an unknown misalignment of the GST set with respect to the spacecraft control coordinate system.

At the conclusion of MESS GST processing, the GST misalignments (possibly including corrected zenith angles) are entered into the SCPS data base. Subsequent spacecraft commands generated by SCPS will then reflect the improved GST alignment parameters.

2.3. The PAX Exercise. The next task in the OAO attitude sensor alignment sequence is to determine the alignment of the pointing axis sensors - BST

and FES - with respect to the self-consistent GST set. This is a fairly simple task. If the spacecraft is held on the GST set in all three axes at an attitude which nominally points the PAX sensors at a suitable target star, each PAX sensor will independently take on errors corresponding directly to its misalignment. Conversely, if the spacecraft is held on one of the PAX sensors - say the FES - in pitch and yaw, and GST in roll, then the GST set will take on gimbal errors due to the offset position of the pointing axis caused by the FES misalignment. In addition, the BST will take on errors corresponding directly to its misalignment relative to the FES.

The PAX Exercise is performed simply by holding on one PAX sensor in pitch and yaw and GST or IRU in roll. Then a single attitude data transmission is sent to the MESS computer. The data of interest in this transmission are the PAX sensor errors and the GST configuration and errors.

When this transmission has been received, MESS PAX processing is initiated. The telemetry data are merged with the spacecraft attitude and characteristics and passed to the PAX components of MESS. These components compute the offset of the GST set from the PAX sensor which is controlling pitch and yaw. The pitch and yaw offsets represent directly the misalignments of that PAX sensor. The errors of the other PAX sensor give its misalignments directly. These misalignments are all then reference transformed to a consistent reference. The reference considered nominal by MESS is defined by the FES in pitch and yaw, and a least-squares fit to the zenith positions of the side-looking GST set in roll. The resulting set of alignment parameters represents a consistent alignment of all the stellar sensors, and provides stellar referenced coordinates within the spacecraft against which the IRU can be aligned.

At the conclusion of MESS PAX processing, the alignment parameter values determined in the PAX Exercise are entered into the data base, to reflect improved alignment of the pointing axis sensors in subsequent commands.

2.4. The IRU Exercise. The last task in the OAO alignment sequence is the determination of the IRU slew axes alignments with respect to the stellar reference coordinates, and the estimation of the IRU slew angle scale factors. The IRU Exercise is the most time-consuming of the MESS/OAO Exercises, and requires the most advance planning. The reason for this state of affairs is that the IRU slew characteristics are invisible to the spacecraft while the spacecraft is holding at a fixed attitude. Only when the spacecraft is commanded to perform a slew about a given nominal IRU axis does it become apparent that the spacecraft has actually slewed about an axis which is slightly misaligned from nominal. And only when the spacecraft is commanded to slew through a given angle does it become apparent that the spacecraft actually has slewed through a slightly different angle, either too large or too small. Thus it takes a slew through a significant angle to generate one data point for the IRU Exercise. The mathematical minimum number of properly defined slews to determine all the IRU parameters is six, but this allows for no statistical redundancy. Twelve slews of proper type and size is considered a minimum number for statistical purposes, and eighteen slews will give significantly better results, and include a margin for operational failures. Since in general only one slew can be performed per contact, it is necessary to set aside an interval of perhaps ten orbits for precise IRU alignment.

Each IRU axis is aligned separately, as follows. To be definite, consider the alignment of the IRU pitch axis. Consider two target stars visible to the

FES, separated by an angle of (say) exactly 20°. There exists a spacecraft attitude at which the FES is pointing to the first target star, and from which a pure pitch slew about the nominal IRU pitch axis of precisely +20° will nominally take the FES directly to the second target star. Now if the spacecraft is initially positioned at the attitude which will do this, and then commanded to perform an IRU pitch of exactly +20°, two things will take place. First of all, the spacecraft will slew off to one side or the other of the target, due to any deviation from nominal in the alignment of the IRU pitch axis. Secondly, the spacecraft will pitch too long or too short, due to any deviation from nominal in the positive pitch scale factor. The deviations of the actual spacecraft terminal attitude after the slew from that predicted on the basis of nominal IRU pitch slew parameters represents error data which can be used to solve for the parameter variations which produced the errors.

With the above preliminary explanation, and continuing with the +20° pitch slew example, the IRU Exercise philosophy can now be presented. In the back orbit prior to some contact at which IRU Exercise data is to be taken, assume the spacecraft is positioned at a stable attitude, at optimum roll (roll angle which gives maximum solar paddle power for specified pointing axis orientation) and pointing to the first target star. A roll slew is commanded out of memory which will bring the spacecraft to the exact attitude required for the Exercise, as explained above. This roll slew is so timed that settling is completed just as the contact begins, in order to minimize the time spent away from optimum roll. As soon into the contact as the spacecraft is commandable, it is placed under stellar control so that it is known to be at the exact planned attitude. Then the IRU is reset and the spacecraft is placed under IRU control. All this

activity is performed quickly, so as to be completed before the +20° pitch slew is commanded out of memory. This command is timed to take place some two minutes into the contact. The spacecraft then proceeds to slew, requiring perhaps four minutes to complete slew and settling. The spacecraft will terminate the slew at an actual attitude which is slightly different from the planned terminal attitude, as explained above. The IRU error registers will now all read zero, since the IRU will have satisfied its slew command. However the stellar sensors will all be reporting some errors, corresponding to the deviation of the actual spacecraft attitude from that planned. Next the spacecraft is placed under stellar control, and allowed to settle out. This places the spacecraft at exactly the planned attitude. Thus the IRU will now have taken on some errors, corresponding directly to the attitude deviation in roll, pitch and yaw. At this time, an attitude data transmission is sent to the MESS computer.

At each contact of sufficient duration, one such slew can be performed. In general, the slews will have been chosen to represent all possible cominations of roll, pitch and yaw slews, positive and negative slews, and small, medium and large slews. The transmissions received from the various contacts are simply stored in a data set until the Execise is completed.

When the last transmission has been received, MESS IRU processing is initiated. The telemetry received is merged with the corresponding spacecraft attitudes and characteristics and passed to the IRU components of MESS. These components determine the size of each slew commanded as well as the slew axis and direction, then compute the coefficients of linear equations relating the unknown misalignment and scale factor parameters to the errors observed.

All such equations for all the slews represented in the data are collected into a linear system of equations, and solved in the least-squares sense for the misalignments and scale factors. The resulting estimated misalignments and scale factors come out already correctly referenced to the spacecraft stellar coordinate reference.

At the conclusion of MESS IRU processing, the alignment parameters and scale factors determined in the IRU Exercise are entered into the SCPS data base, so that subsequent slew commands generated by SCPS will bring the spacecraft more precisely to the desired attitudes.

3. MESS System Architecture

MESS/OAO is a 100K-instruction component of the 2000K-instruction Support Computer Program System (SCPS). SCPS is the software which makes flying the OAO possible. The OAO must be continually supplied with fresh commands, 24 hours a day, as much just to keep it stable and under control as to enable it to collect scientifically useful data. To do this, SCPS takes inputs from the mission and scientific staffs, and fabricates contact messages containing command memory loads which will keep the OAO functioning properly until the follow-on memory load is received. This process has been repeated, many times a day, for over 2 1/2 years now to keep OAO-A2 alive and well.

SCPS resides on a dedicated IBM 360/65 system with 1.5M bytes of core. MESS/OAO, an off-line component of SCPS, has an overlay tree structure, schematically represented in Figure 2, which allows it to fit within a maximum of 450K bytes of core. The root segment of MESS primarily consists of the

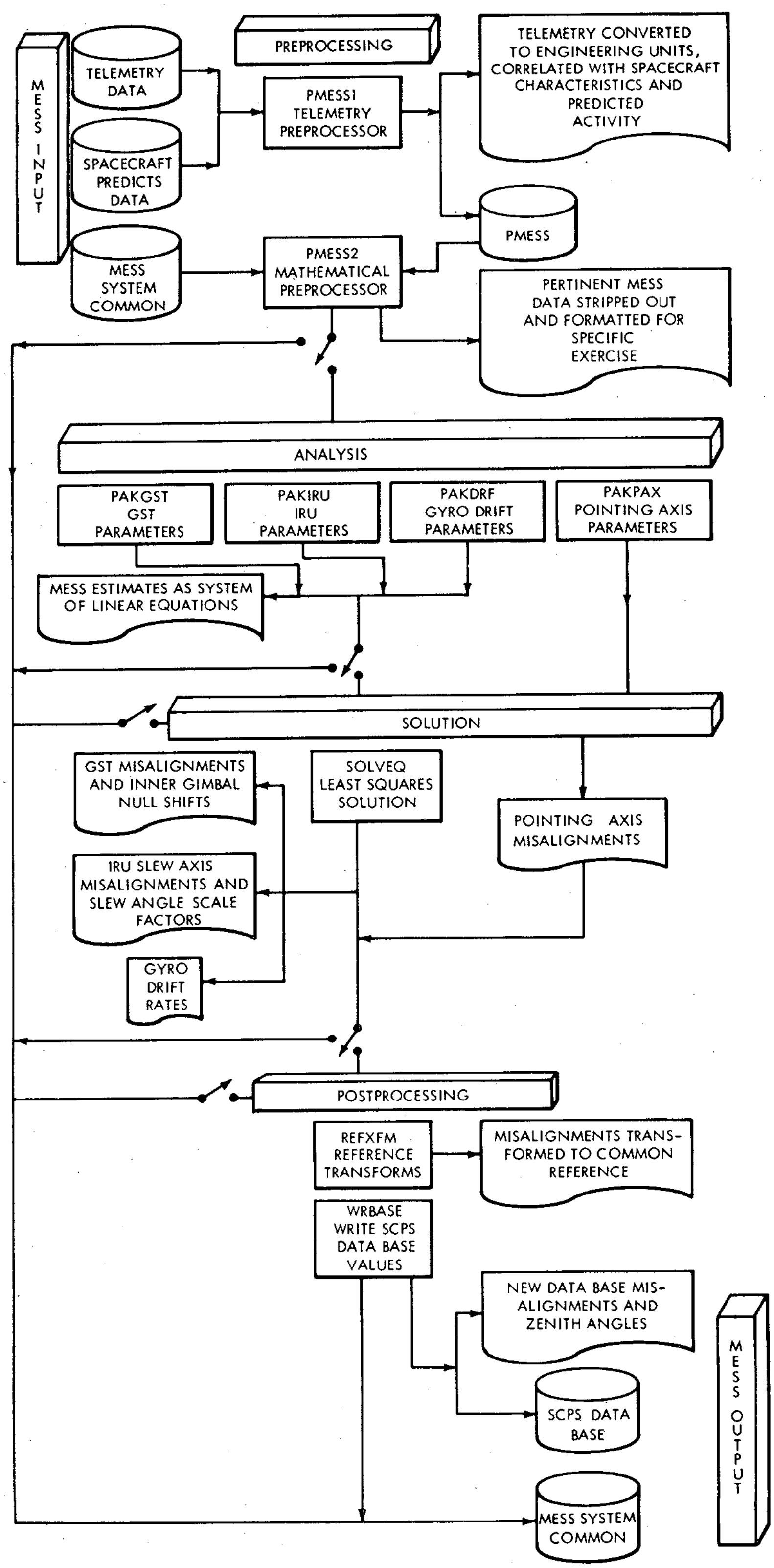


Figure 2. MESS Structure

Driver, which invokes paths specified in the control record input stream. The overlay segments correspond to the functional areas of Preprocessing, Analysis, Solution and Postprocessing. Any MESS run may be interrupted or a previous run resumed at any point. Disk common preserves continuity between runs.

Sequencing through the MESS subroutines is controlled by records placed in an input control stream. This stream is normally written on a 2260 display device, with card backup. The control stream is also used to make changes to MESS System Common between subroutine executions so as to provide for several consecutive runs against the same data, using different run parameters. A summary of all significant MESS output is provided at the 2260 display station, so that the analyst may have immediate knowledge of results and can direct subsequent runs accordingly. An entire MESS alignment episode can be monitored, controlled and validated by the MESS Analyst from the display station.

3.1. Preprocessing. There are two MESS preprocessors - PMESS1, the telemetry preprocessor, and PMESS2, the mathematical preprocessor.

3.1.1. PMESS1, the telemetry preprocessor, selects and merges specified data from the data sets containing the actual returned telemetry data, and the spacecraft model and activity predicts data. The telemetry data are first edited, averaged and weighted, according to options specified in the control stream, and converted to engineering units. The spacecraft model and activity predicts data are then searched for those data pertaining to the time contained in the telemetry data. When a match is obtained, the pertinent spacecraft characteristics and predicted attitude are then stripped out of the predicts data and merged with the telemetry data. The end result is the temporary storage data set PMESS, which

contains a header record of spacecraft characteristics, and additional records of merged and edited attitude and telemetry data.

3.1.2. PMESS2, the mathematical preprocessor, then reads the PMESS data set and pulls out those data pertinent to the type of Exercise - PAX, GST, IRU or DRF - which was performed. These data are then transformed to the model definitions, and formatted into arrays suitable for direct input to the analysis subroutine to be used.

3.2. Analysis. Next, the pertinent analysis subroutine is called. There is a different one of these for each different Exercise. These subroutines contain the mathematical analysis of the problem; they all function by analyzing the data and computing the elements of arrays which can subsequently be processed by a strictly mathematical algorithm to yield the solution estimates directly.

3.2.1. PAKDRF is used to estimate the IRU gyro drift rates. These rates are derived with respect to specific GST patterns. That is, the IRU errors are read once or more per orbit while the spacecraft is holding on a particular GST pattern. As the IRU platform gradually drifts away from the stellar reference, the IRU errors build up, and the rate of buildup is estimated by MESS. In practice, various patterns are used as they become available around the orbit, and PAKDRF automatically sorts the data by pattern, so as to derive independent estimates of rate for each pattern (up to a maximum of five). The DRF Exercise is normally performed prior to any of the other MESS Exercises, so as to provide a nondrifting spacecraft as required for portions of the subsequent Exercises. The output from PAKDRF takes the form of a set of linear equations which can subsequently be solved to give the gyro drift rate estimates directly.

3.2.2. PAKGST is used to align the gimballed star trackers. These sensors are aligned only relatively - that is, they are aligned to one another so as to be self-consistent. This Exercise will normally be performed prior to the PAX and IRU Exercises, so that the misalignments determined during these latter Exercises will be referenced to a self-consistent GST set. The output from PAKGST takes the form of a set of linear equations which can subsequently be solved to give the raw GST misalignments directly.

3.2.3. PAKPAX is used to align the pointing axis sensors - the boresighted star tracker (BST) and the GEP fine error sensor (FES). These sensors are aligned independently of each other to a reference system defined by the GST. This Exercise is normally performed prior to the IRU Exercise so that the slew axis misalignments determined during that Exercise will be referenced to an internally self-consistent set of stellar sensors. PAKPAX contains its own solution algorithm, and outputs the raw BST and FES misalignment estimates directly.

3.2.4. PAKIRU is used to estimate the slew axis misalignments and slew angle scale factors. That is, if slews in a certain direction are terminating consistently off to one side or too long or short, PAKIRU will estimate the values of the parameters which describe such deviations. The IRU is aligned to a reference system defined by the best available stellar sensors (ideally, the FES in pitch and yaw, and GST in roll). The output from PAKIRU takes the form of a set of linear equations which can subsequently be solved to give the estimates of the raw slew axis misalignments and slew angle scale factor deviations directly.

3.3. Solution. One solution subroutine is used to solve the linear systems constructed by the subroutines PAKGST, PAKIRU and PAKDRF. (PAKPAX contains its own solution algorithm.)

3.3.1. SOLVEQ is a general least-squares algorithm for solving systems of up to 300 equations which are over-determined (more linearly independent equations than unknowns) in up to 30 unknowns. The algorithm takes as input a coefficient matrix C(300, 30) and a right-hand-side vector E(300), as well as a row mask MASKR(300) and a column mask MASKC(30). The algorithm solves the equation CX = E where X(30) is the array of solution values, according to a least-squares criterion, restricted by the masks as follows.

Stated informally, SOLVEQ takes up to 300 equations linear in up to 30 unknowns, assumes specified values of some selected unknowns, and then solves (in the least-squares sense) selected equations in selected remaining unknowns. More precisely, the algorithm selects the rows i of C specified by the values MASKR(i) = 1 (MASKR will usually be set so that these are the nonzero rows of C). Then the algorithm sets solution values for any parameters X(j) specified by MASKC(j) = 2. The algorithm then solves in the least-squares sense the equations selected by MASKR(i) = 1 for the unknowns X(j) selected by MASKC(j) = 1, assuming the values X(j) selected by MASKC(j) = 2.

The output from SOLVEQ consists of the solution array X. In the event the masked system is rank-deficient (fewer linearly independent equations than unknowns), SOLVEQ outputs diagnostic information to identify the source of the deficiency.

3.4. Postprocessing. There are two MESS postprocessors - REFXFM, which reference transforms the various raw misalignment estimates to a consistent reference, and WRBASE, which writes out the revised parameter values for the SCPS data base.

3.4.1. REFXFM contains models of all the various coordinate systems to which the misalignments could conceivably be referenced. References for roll, pitch, yaw and the IRU are selected by input parameters IROLL, IPITCH, IYAW and IIRU. Misalignments in roll, pitch and yaw can be referenced to any appropriate individual attitude sensor, or to an appropriate least-squares combination of GST. The IRU misalignments can be further separated into sets of orthogonal and non-orthogonal misalignments, or combined. The output from REFXFM is a consistent set of alignment parameter estimates for all the sensors involved in the preceding Exercises. It is in REFXFM that the results of Exercises for separate sensors are combined into a consistent set of estimates.

3.4.2. WRBASE takes the consistent set of misalignment and calibration parameter estimates from REFXFM, and combines it with the nominal SCPS values (current data base values) to produce a revised set of values for the SCPS data base. The output from WRBASE is a table of values in a form suitable for direct entry to members IDL005 and IDL009 of the data base, when validated.

3.5. MESS Common. All MESS subroutines are (with trivial exceptions) argument-free, and pass their inputs and outputs through MESS Systems Common. All variables in MESS Common have been assigned unique names, and all these names have been listed in a NAMELIST statement in the MESS Driver.

Thus all MESS variables are accessible to the external world via NAMELIST read and write statements.

This capability is used primarily to change the values of selected variables between calls to MESS subroutines. Thus it is possible in effect to alter the MESS run dynamically during execution, provided the changes desired can be predefined. For example, a given set of MESS Exercise data can be processed under a series of different options, all within the same execution cycle. The first run is made, then the Common variables are changed so as to set up the second run, then the second run is made, and so on. The changes to MESS Common, as well as the sequence of subroutines to be called, are defined by an input stream of control records.

4. Mathematical Analysis

A brief sketch of the core analysis is given herein. For more details, consult the References. The comprehensive MESS Manual, to be published shortly, will be a compendium of all published material on MESS.

4.1. The Spacecraft Model. For purposes of this discussion, a spacecraft in orbit stabilized in three axes will be represented by a right-handed orthogonal control coordinate system c being maintained at attitude A relative to the usual geocentric inertial coordinate system o. That is, direction cosines $\vec{v}$ of any direction fixed in inertial space o are transformed by an attitude matrix A into direction cosines $A\vec{v}$ expressed in control coordinates c. From control coordinates, the vector $A\vec{v}$ must undergo the transformation R_s to be expressed in the (known) local coordinate system s of attitude sensor s. From this local

coordinate system, the vector $R_s A\vec{v}$ must be transformed by the modeled misalignments $I + dR_s$, where I is the identity, to be expressed in the (unknown) misaligned coordinate system s'. Figure 3 shows the flow diagram of these coordinate systems for a typical sensor s. Additional sensors t, u, . . ., may also be referenced to the control coordinate system, as shown in Figure 3.

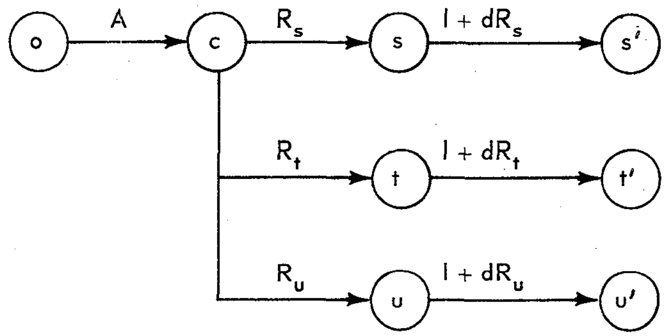


Figure 3. Attitude Sensor Coordinate Transforms

What is included in R_s depends on one's point of view. For the present discussion, R_s may be considered the nominal transformation (as determined by spacecraft design), while $I + dR_s$ represents the total misalignment. In the operational environment, some values for the misalignment parameters will already have been determined by preflight measurement or previous inflight estimation. In these cases, R_s may be taken to represent the misaligned coordinate system specified by those values of the parameters, while $I + dR_s$ represents residual misalignments which remain to be calibrated out.

The exact form of R_s depends on the location of sensor s relative to the control coordinates c, and will be unique to the particular spacecraft. Transformations used in MESS/OAO are given in appropriate later sections.

The attitude A of the spacecraft is defined by a yaw-pitch-roll Euler angle sequence. The control coordinate system axes c are obtained from the inertial coordinate system axes o by rotating the latter first by a yaw (positive sense about positive z-axis) through right ascension α, next by a negative pitch (negative sense about positive y-axis) through declination δ, finally by a roll (positive sense about positive x-axis) through roll angle β. In the matrix formulation,

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C\beta & S\beta \\ 0 & -S\beta & C\beta \end{pmatrix} \begin{pmatrix} C\delta & 0 & S\delta \\ 0 & 1 & 0 \\ -S\delta & 0 & C\delta \end{pmatrix} \begin{pmatrix} C\alpha & S\sigma & 0 \\ -S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} C\delta\, C\alpha & C\delta\, S\alpha & S\delta \\ -C\beta\, S\alpha - S\beta\, S\delta\, C\alpha & C\beta\, C\alpha - S\beta\, S\delta\, S\alpha & S\beta\, C\delta \\ S\beta\, S\alpha - C\beta\, S\delta\, C\alpha & -S\beta\, C\alpha - C\beta\, S\delta\, S\alpha & C\beta\, C\delta \end{pmatrix}$$

(C = cos, S = sin), so that $\vec{x}_c = A\vec{x}_o$. Conversely, given the attitude matrix A, one can determine the right ascension α, declination δ and roll angle β which it represents by the equations

$$\alpha = \mathrm{ATAN2}\ (A_{12},\ A_{11})$$

$$\delta = \mathrm{ATAN}\ \frac{A_{13}}{\sqrt{A_{11}^2 + A_{12}^2}}$$

$$\beta = \mathrm{ATAN2}\ (A_{23},\ A_{33})$$

4.2. Analysis for the DRF Exercise. The DRF Exercise involves only classic least-squares analysis. For each given pattern, suppose n data points have been collected. These data have the form $(t_i, \vec{e}_i)$, i = 1, 2, . . ., n, where t_i is the time the i^{th} data point was collected, and $\vec{e}_i$ is the corresponding array of IRU gyro errors in roll, pitch and yaw. The errors are modeled as growing linearly with time. Hence, it is assumed that there exist unknown constant arrays $\vec{a}$, $\vec{b}$, such that

$$\vec{e}_i = \vec{a}\, t_i + \vec{b}, \quad i = 1, 2, \cdots, n$$

Expressing these as 3n scalar equations in 6 unknowns, we can write

$$e_{ij} = a_j\, t_i + b_j, \quad i = 1, 2, \cdots, n, \quad j = 1, 2, 3$$

Each of these equations can be considered as expressing the error e_{ij} as a linear function of the unknowns a_j, b_j:

$$e_{ij} = t_i\, a_j + 1 \cdot b_j, \quad i = 1, 2, \cdots, n, \quad j = 1, 2, 3$$

Thus the 3n equations can be expressed in the following matrix form CX = E:

$$\begin{bmatrix} t_1 & 1 & 0 & 0 & 0 & 0 \\ t_2 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t_n & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & t_1 & 1 & 0 & 0 \\ 0 & 0 & t_2 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & t_n & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_1 & 1 \\ 0 & 0 & 0 & 0 & t_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & t_n & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} e_{11} \\ e_{21} \\ \vdots \\ e_{n1} \\ e_{12} \\ e_{22} \\ \vdots \\ e_{n2} \\ e_{13} \\ e_{23} \\ \vdots \\ e_{n3} \end{bmatrix}$$

In general, the data $(t_i, \vec{e}_i)$ are inconsistent, so this equation is overdetermined in the parameters $\vec{a}$, $\vec{b}$. In MESS, the least-squares solution is chosen, i. e., the solution vector X is found which minimizes $||CX - E||$. The solution parameters $\vec{a}$ represent the useful MESS output; the solution values of a_1, a_2, a_3 are the gyro error rate estimates in roll, pitch and yaw, respectively.

4.3. Analysis for the GST Exercise. The six gimballed startrackers on OAO-A2 are oriented one at each end of the three coordinate axes, as shown in Figure 4. Local tracker coordinate systems x_k, y_k, z_k, $k = 1, 2, \ldots, 6$, are defined such that the zenith position of tracker k lies along the $+x_k$-axis, the inner gimbal axis at zenith coincides with the y_k-axis, and the outer gimbal axis coincides with the z_k-axis. Within the local tracker k coordinate system, the outer and inner gimbal phasing is defined according to the right ascension-declination convention: outer gimbal motion about the z_k-axis is positive counterclockwise, inner gimbal motion about the inner gimbal axis is positive clockwise. These relationships are shown in Figure 5.

Hence a star in the field of view of tracker k with gimbal angles σ_k, μ_k (outer, inner, resp.) has local coordinates

$$\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} = \begin{pmatrix} c\sigma_k & s\sigma_k & 0 \\ -s\sigma_k & c\sigma_k & 0 \\ 0 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} c\mu_k & 0 & s\mu_k \\ 0 & 1 & 0 \\ -s\mu_k & 0 & c\mu_k \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} c\sigma_k \; c\mu_k \\ s\sigma_k \; c\mu_k \\ s\mu_k \end{pmatrix}$$

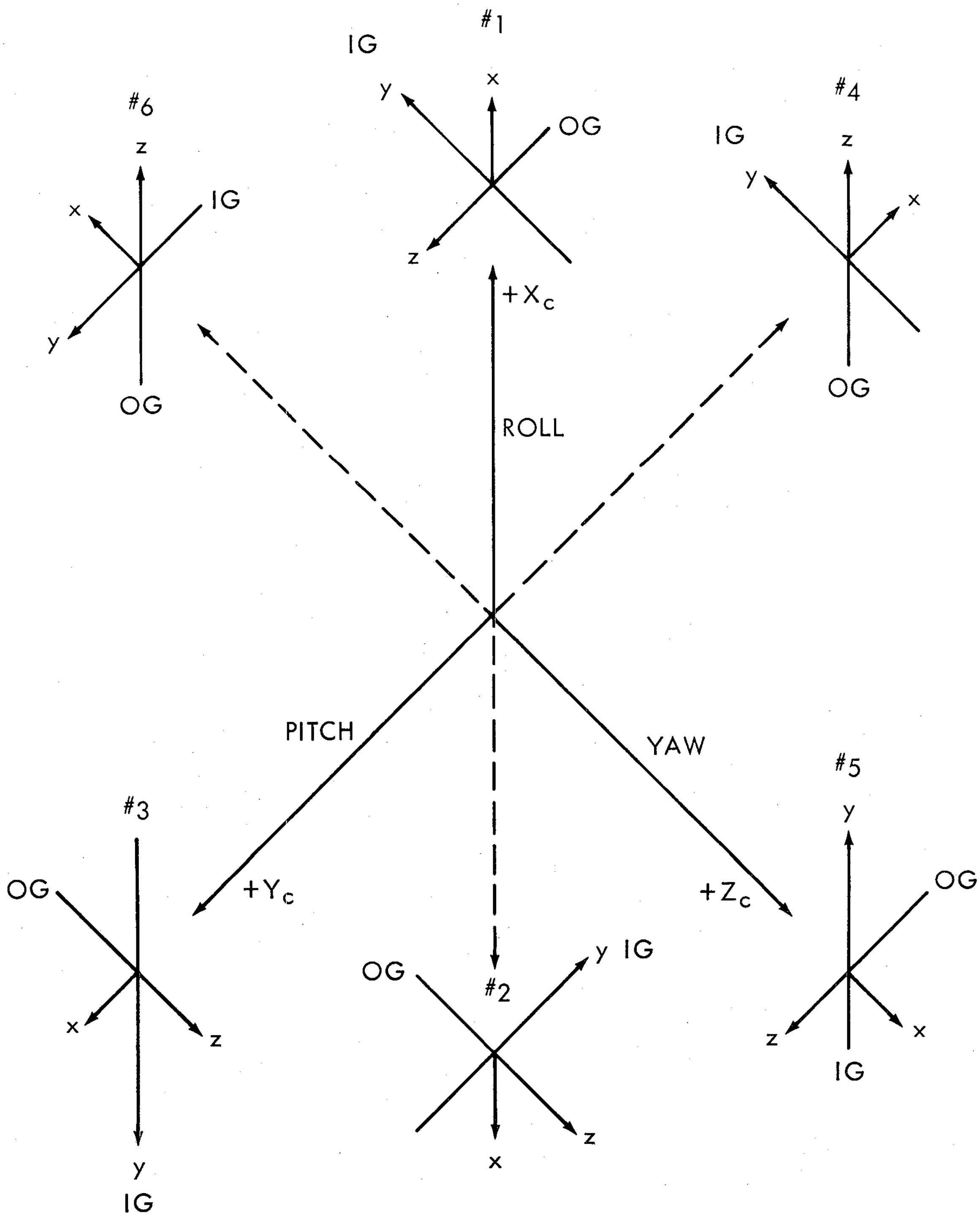
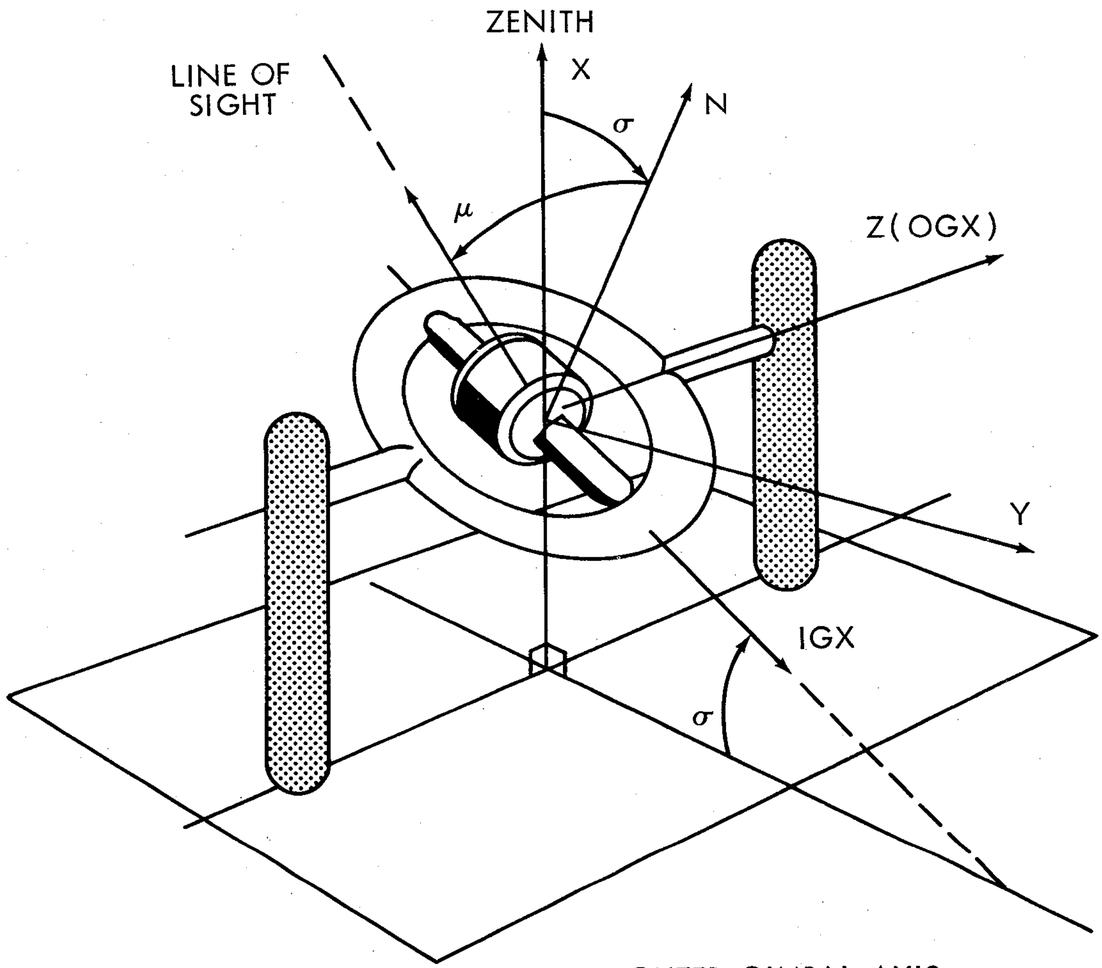


Figure 4. Control Axis System and Star Tracker Gimbal Locations



OGX	=	OUTER GIMBAL AXIS
σ	=	OUTER GIMBAL ANGLE
IGX	=	INNER GIMBAL AXIS
μ	=	INNER GIMBAL ANGLE
N	=	NORMAL TO THE GIMBAL PLANE (the plane determined by the two gimbal axes)

Figure 5. Gimbal Angles

This transformation is represented by the following coordinate-transformation diagram:

$$\textcircled{k} \xrightarrow[z]{\sigma_k} \bigcirc \xrightarrow[-y]{\mu_k} \textcircled{k^*}$$

As each of the six local coordinate systems is nominally aligned parallel to some control set of axes, the nominal transformation R_k from the control axis c to the system k has a matrix composed only of 0, ±1:

$$R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad R_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_4 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad R_6 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Each such transformation is represented by the following diagram:

$$\textcircled{c} \xrightarrow{R_k} \textcircled{k}$$

The misalignments modeled were, first, rotational misalignments $d\phi_k$, $d\theta_k$, $d\psi_k$ (taken positive in the conventional right-hand sense) about each of the tracker k coordinate axes x_k, y_k, z_k, resp. These represent an arbitrary misalignment of the tracker k gimbal platform relative to the spacecraft structure

as a whole. Making the usual small-angle (first-order) approximations

$$c\alpha \approx 1, \quad s\alpha \approx \alpha \; (|\alpha| \ll 1)$$

the misalignments can be represented by a small-angle rotation matrix

$$I + d\Phi_k = \begin{pmatrix} 1 & d\psi_k & -d\theta_k \\ -d\psi_k & 1 & d\phi_k \\ d\theta_k & -d\phi_k & 1 \end{pmatrix}$$

Second, there were also modeled shifts of the null position in the inner and outer gimbals. A null shift in the outer gimbal cannot be distinguished from a misalignment about the tracker z_k-axis, since the outer gimbal axis is always parallel to the z_k-axis. Hence no separate parameter is necessary to represent this shift.

However, a null shift $d\beta_k$ in the inner gimbal can be separated from a misalignment about the tracker y_k-axis by taking a large outer gimbal angle, since this separates the inner gimbal axis from the y_k-axis. (The inner gimbal axis is parallel to the y_k-axis only when the outer gimbal axis is zero.)

Because $d\beta_k$ masquerades as $d\theta_k$ for zero outer gimbal angle, the sense of $d\beta_k$ has been taken as that of $d\theta_k$, viz., positive in the usual right-hand sense. This is <u>opposite</u> to the sense of the inner gimbal angle itself, which is that of declination (declination is negative in the usual right-hand sense).

Thus we have defined the following coordinate transformation models for each tracker. In the nominal case, the outer and inner gimbal angles σ_k, μ_k, resp., are computed based on the known attitude of the spacecraft and the known misalignments $d\Omega_k$:

$$c \xrightarrow{R_k} k \xrightarrow{I + d\Omega_k} k' \xrightarrow[Z]{\sigma} \bigcirc \xrightarrow[-y]{\mu_k} k^*$$

These gimbal angles would nominally point the startracker k line-of-sight directly at the target star. Due to unknown misalignments $d\Phi_k$, $d\beta_k$, however, and to the fact that the true spacecraft attitude could be somewhat removed from nominal, the tracker will not in general find the target star at the <u>commanded</u> angles σ_k, μ_k, but rather at <u>measured</u> angles σ'_k, μ'_k slightly different from σ_k, μ_k. Hence including misalignments, we have the following situation:

$$c \xrightarrow{R_k} k \xrightarrow{I + d\Omega_k} k' \xrightarrow{I + d\Phi_k} k'' \xrightarrow[z]{\sigma'_k} \bigcirc \xrightarrow[-y]{\mu'_k - d\beta_k} k^*$$

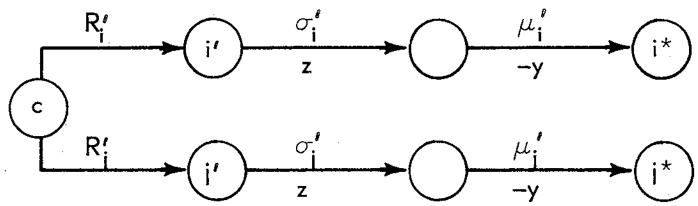
(The negative sign before $d\beta_k$ is due to the fact that its sense is opposite to that of μ'_k.) Thus for a given star being tracked by a given tracker, we have the following:

$$\begin{tikzcd}
& & \bigcirc \arrow[rrd, "\mu_k", "-y"'] & & \\
c \arrow[r, "R'_k"] & k' \arrow[ru, "\sigma_k", "z"'] \arrow[rd, "I + d\Phi_k"'] & & & k^* \\
& & k'' \arrow[r, "\sigma'_k", "z"'] & \bigcirc \arrow[ru, "\mu'_k - d\beta_k", "-y"'] &
\end{tikzcd}$$

where $R'_k = (I + d\Omega_k)\, R_k$.

Now suppose the spacecraft is holding at some fixed attitude, and consider the tracking startrackers by pairs. On the one hand, the angle between two stars individually being tracked by two corresponding startrackers is known precisely

from star catalogs. On the other hand, the angle between the tracking startrackers as computed from the gimbal measurements will differ from the true angle, and this discrepancy is assumed to be due to misalignments of the trackers involved. Consider the following diagram:



The computed dot product between the trackers i and j is given by

$$b' = \vec{s}_j^{\,\prime c} \cdot \vec{s}_i^{\,\prime c}$$

where $\vec{s}_i^{\,\prime c}$ are the measured (') coordinates of star i ($\vec{s}_i$) in the control coordinate system (^{c}). From the diagram:

$$\vec{s}_i^{\,\prime c} = R_i'^{T} \begin{pmatrix} c\sigma_i' \; c\mu_i' \\ s\sigma_i' \; c\mu_i' \\ s\mu_i' \end{pmatrix}$$

Similarly for tracker j.

On the other hand, the true dot product between the stars is known from the star catalog, and may be expressed as a function of the (unknown) misalignments and the (known) measured angles:

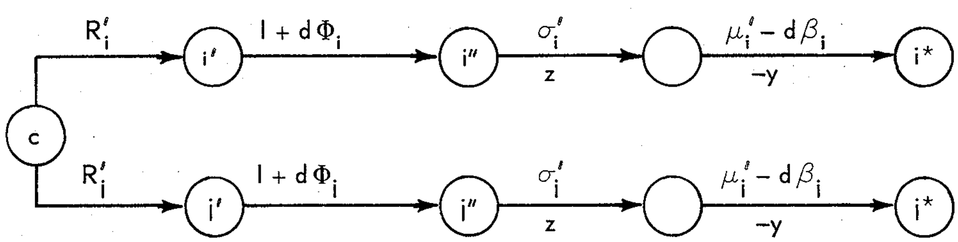


Figure 6

Taking the dot product in the i' coordinate system:

$$b = \vec{s}_j^{\,i'} \cdot \vec{s}_i^{\,i'} .$$

The difference $\Delta b \equiv b - b'$ is a function of the eight unknown misalignments $d\phi_i$, $d\theta_i$, $d\psi_i$, $d\beta_i$, $d\phi_j$, $d\theta_j$, $d\psi_j$, $d\beta_j$, and hence may be expressed to first order in differential form:

$$\Delta b = \left[\left(\frac{\partial \Delta b}{\partial d\phi_i} \right)_0 \left(\frac{\partial \Delta b}{\partial d\theta_i} \right)_0 \cdots \left(\frac{\partial \Delta b}{\partial d\beta_j} \right)_0 \right] \begin{bmatrix} d\phi_i \\ d\theta_i \\ \vdots \\ d\beta_j \end{bmatrix}$$

where the differential coefficients are all evaluated in the nominal state, i. e., assuming zero misalignments. For any given data reading, the discrepancy $\Delta b = b - b'$ is computed from the known coordinates of the stars and the data (the measured angles σ_i', μ_i', σ_j', μ_j'). The differential coefficients are also computed from the data, as follows:

$$\Delta b = b - b' = \vec{s}_j^{\,i'} \cdot \vec{s}_i^{\,i'} - \vec{s}_j^{\,\prime c} \cdot \vec{s}_i^{\,\prime c} ,$$

$$\frac{\partial \Delta b}{\partial d\phi_i} = \frac{\partial}{\partial d\phi_i} \left(\vec{s}_j^{\,i'} \cdot \vec{s}_i^{\,i'} \right) = \vec{s}_j^{\,i'} \cdot \frac{\partial \vec{s}_i^{\,i'}}{\partial d\phi_i} .$$

Now

$$\vec{s}_i^{\,i'} = \begin{pmatrix} 1 & d\psi_i & -d\theta_i \\ -d\psi_i & 1 & d\phi_i \\ d\theta_i & -d\phi_i & 1 \end{pmatrix}^T \vec{s}_i^{\,i''} .$$

(The coordinate system i'' is that of the misaligned startracker i gimbal platform. Cf. Figure 6). Hence

$$\frac{\partial \vec{s}_i^{\,i'}}{\partial d\phi_i} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \vec{s}_i^{\,i''} .$$

Evaluating $\vec{s}_i^{\,i'}$ in the nominal state, this becomes

$$\left(\frac{\partial \vec{s}_i^{\,i'}}{\partial d\phi_i}\right)_0 = \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i' \;\; c\mu_i' \end{pmatrix} .$$

Hence

$$\left(\frac{\partial \Delta b}{\partial d\phi_i}\right)_0 = \begin{pmatrix} c\sigma_j' \;\; c\mu_j' \\ s\sigma_j' \;\; c\mu_j' \\ s\mu_j' \end{pmatrix}^T R_j' \, R_i'^T \begin{pmatrix} 0 \\ -s\mu_i' \\ s\sigma_i' \;\; c\mu_i' \end{pmatrix} .$$

In a similar way,

$$\left(\frac{\partial \Delta b}{\partial d\theta_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} s\mu_i' \\ 0 \\ -c\sigma_i' \;\; c\mu_i' \end{pmatrix}, \quad \left(\frac{\partial \Delta b}{\partial d\psi_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} -s\sigma_i' \;\; c\mu_i' \\ c\sigma_i' \;\; c\mu_i' \\ 0 \end{pmatrix} .$$

Finally,

$$\frac{\partial \vec{s}_i^{\,i'}}{\partial d\beta_i} = \begin{pmatrix} 1 & -d\psi_i & d\theta_i \\ d\psi_i & 1 & -d\phi_i \\ -d\theta_i & d\phi_i & 1 \end{pmatrix} \begin{pmatrix} c\sigma_i' \;\; s\mu_i' \\ s\sigma_i' \;\; s\mu_i' \\ -c\mu_i' \end{pmatrix} .$$

At zero misalignments,

$$\left(\frac{\partial \Delta b}{\partial d\beta_i}\right)_0 = \vec{s}_j^{\,i'} \cdot \begin{pmatrix} c\sigma_i' \; s\mu_i' \\ s\sigma_i' \; s\mu_i' \\ -c\mu_i' \end{pmatrix}.$$

The coefficients

$$\left(\frac{\partial \Delta b}{\partial d\phi_j}\right)_0, \quad \left(\frac{\partial \Delta b}{\partial d\theta_j}\right)_0, \quad \left(\frac{\partial \Delta b}{\partial d\psi_j}\right)_0, \quad \left(\frac{\partial \Delta b}{\partial d\beta_j}\right)_0$$

can be computed using the above formulas, due to the symmetry of i and j, simply by interchanging i and j. Hence the following equation in eight unknowns has been generated:

$$[C_{ji} \,|\, C_{ij}] \begin{bmatrix} d\phi_i \\ d\theta_i \\ d\psi_i \\ d\beta_i \\ d\phi_j \\ d\theta_j \\ d\psi_j \\ d\beta_j \end{bmatrix} = b - b'$$

where C_{ji}, C_{ij} are 1×4 matrices:

$$C_{k\ell} = \begin{pmatrix} c\sigma_k' \; c\mu_k' \\ s\sigma_k' \; c\mu_k' \\ s\mu_k' \end{pmatrix}^T R_k' \, R_\ell'^T \begin{pmatrix} 0 & s\mu_\ell' & -s\sigma_\ell' \; c\mu_\ell' & c\sigma_\ell' \; s\mu_\ell' \\ -s\mu_\ell' & 0 & c\sigma_\ell' \; c\mu_\ell' & s\sigma_\ell' \; s\mu_\ell' \\ s\sigma_\ell' \; c\mu_\ell' & -c\sigma_\ell' \; c\mu_\ell' & 0 & -c\mu_\ell' \end{pmatrix}$$

Altogether in the system of the six gimballed startrackers, there are 24 unknowns, and the second above equation may be regarded as one equation in 24 unknowns. Each pair of tracking startrackers generates one such equation for each data reading. A set of three tracking startrackers taken by pairs generates three such equations, and in general a set of n tracking startrackers generates $\binom{n}{2}$ such equations (some redundant).

Data readings are collected representing many different values of σ, μ for all trackers, and the equations described above are generated. In this way a large number of equations in the 24 unknown misalignments are generated. This system of equations is then solved in the least-squares sense. The solutions represent the least-squares estimates sought.

4.4. Analysis for the PAX Exercise. Consider a specified attitude A_o at which the nominal spacecraft control coordinate system c has its $+x_c$-axis directed to a specified FES target. At this attitude, outer and inner gimbal angles σ_i, μ_i, $i = 1, 2, \ldots n$, are sent to the spacecraft to point the n tracking GST to their respective guide stars. In the spacecraft control coordinate system, then, the n guide stars are located by the vectors

$$\vec{v}_i = R_i^T (I + d\Phi_i)^T \begin{pmatrix} C\sigma_i \ C\mu_i \\ S\sigma_i \ C\mu_i \\ S\mu_i \end{pmatrix}, \quad i = 1, 2, \cdots, n$$

where C = cosine, S = sine, $d\Phi_i$ is the misalignment matrix

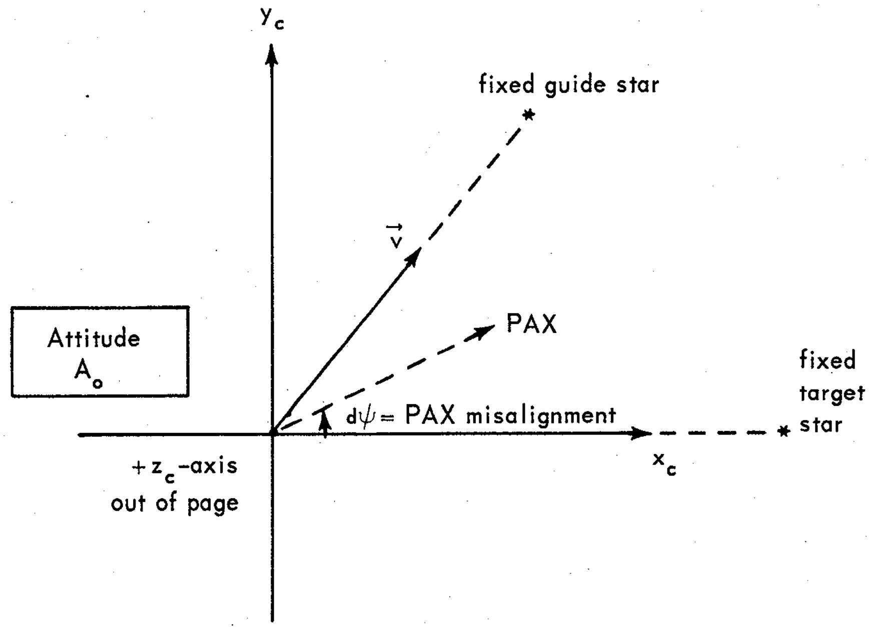
$$\begin{pmatrix} 0 & d\psi_i & -d\theta_i \\ -d\psi_i & 0 & d\phi_i \\ d\theta_i & -d\phi_i & 0 \end{pmatrix}$$

and R_i is the matrix relating the nominal tracker i local coordinate system to the control coordinate system c.

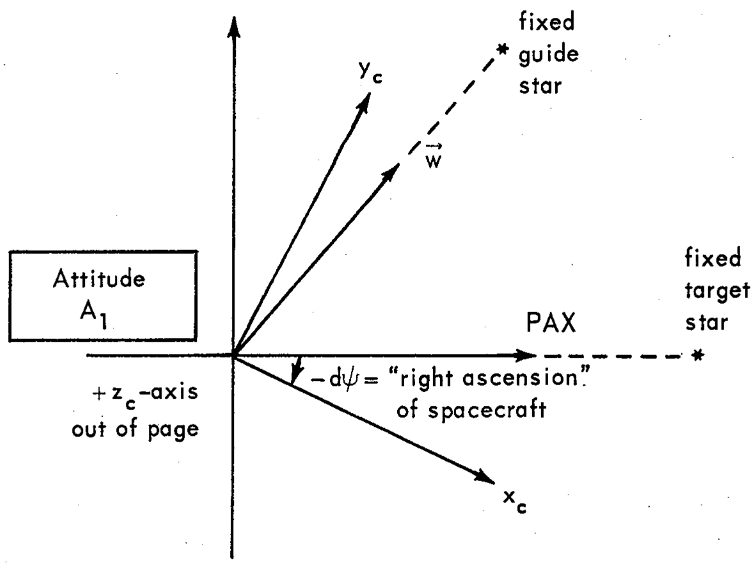
When a pointing axis sensor PAX is allowed to assume control of pitch and yaw, its misalignments $d\theta$ in pitch and $d\psi$ in yaw cause the spacecraft to move in the directions and magnitudes of the negatives of the above misalignments. Thus the spacecraft will move in the positive sense about the $+y_c$ -axis through the angle $-d\theta$, and in the positive sense about the $+z_c$-axis through the angle $-d\psi$. The spacecraft will thus take up a new attitude A_1. At this attitude, the n tracking GST will take on outer and inner gimbal errors $\Delta\sigma_i, \Delta\mu_i$, i = 1, 2, . . ., n. Thus in the new position of the control coordinate system, the n guide stars are located by the vectors

$$\vec{w}_i = R_i^T (I + d\Phi_i)^T \begin{pmatrix} C(\sigma_i + \Delta\sigma_i) \; C(\mu_i + \Delta\mu_i) \\ S(\sigma_i + \Delta\sigma_i) \; C(\mu_i + \Delta\mu_i) \\ S(\mu_i + \Delta\mu_i) \end{pmatrix}, \quad i = 1, 2, \cdots, n$$

The rotation which the spacecraft undergoes in moving from attitude A_0 to attitude A_1 can be determined by finding the rotation matrix R which will move the vectors $\{\vec{v}_i\}$ into the vectors $\{\vec{w}_i\}$ in the sense of making $R\,\vec{v}_i = \vec{w}_i$, i = 1, 2, . . ., n. This rotation represents the "attitude" of the new position of the spacecraft in the coordinate system defined by the old position of the spacecraft control coordinate system c. The following "before" and "after" figures may be instructive.



y_c
fixed guide star
$\vec{v}$
PAX
Attitude A_o
fixed target star
$d\psi$ = PAX misalignment
x_c
$+z_c$-axis out of page



fixed guide star
y_c
$\vec{w}$
Attitude A_1
fixed target star
PAX
$-d\psi$ = "right ascension" of spacecraft
$+z_c$-axis out of page
x_c

From these figures and matrix analysis, it can be seen that the rotation R which satisfies $R\,\vec{v} = \vec{w}$ has the form

$$R = \begin{pmatrix} Cd\psi & -Sd\psi \\ Sd\psi & Cd\psi \end{pmatrix}$$

This represents an "attitude" with "right ascension" $-d\psi$.

An SCPS computer program - DOAOP - is used by MESS to compute the rotation matrix M which minimizes the function

$$f\,(M) = \sum_{i=1}^{n} \|M\,\vec{v}_i - \vec{w}_i\|^2$$

for given input vectors $\vec{v}_i$, $\vec{w}_i$, $i = 1, 2, \ldots, n$. Thus DOAOP can be used to find the rotation matrix which "best" (in the sense of least squares) estimates the attitude change required above. For small pitch and yaw misalignments $d\theta$, $d\psi$, resp., the attitude matrix returned by DOAOP will have the nominal form

$$\begin{pmatrix} 1 & -d\psi & d\theta \\ d\psi & 1 & 0 \\ -d\theta & 0 & 1 \end{pmatrix}$$

4.5. <u>Analysis for the IRU Exercise.</u> The IRU is designed to provide precise slew sensing about each of the spacecraft control coordinate axes in either the positive or negative direction (taken in the right-hand sense). First-order deviations in slew-axis alignment are provided in the SCPS spacecraft model by the matrix

$$I + d\Omega = \begin{pmatrix} 1 & d\omega_{12} & d\omega_{13} \\ d\omega_{21} & 1 & d\omega_{23} \\ d\omega_{31} & d\omega_{32} & 1 \end{pmatrix}$$

The nominal values of the off-diagonal elements $d\omega_{ij}$, $i \neq j$, in the above matrix are all zeroes. The i^{th} column ($i = 1, 2, 3$) of the above matrix is a first-order unit vector giving the actual (misaligned) i^{th} slew axis (1 = roll, 2 = pitch, 3 = yaw) represented in the control coordinate system.

First-order deviations in slew-angle sensing are provided in the SCPS spacecraft model by slew angle scale factor calibration parameters. There are six such, one each for the positive and negative directions of slew about the three slew axes. These parameters are taken to be small corrections ζ_{ij} which have to be added to the nominal scale factor 1 to produce the correct scale factor for the slew angle. That is, if a slew of angle θ is commanded about the i-axis (1 = roll, 2 = pitch, 3 = yaw) in the j-direction (1 = positive, 2 = negative), the slew which actually occurs has angle $(1 + \zeta_{ij})\,\theta$.

The approach taken in the IRU Exercise analysis is to construct a system of linear equations which relate the small unknown changes in the values of the alignment and calibration parameters, to the small errors in terminal slew attitude which result from those changes. If a large number of such equations is available, the unknown changes in parameter values may be estimated by standard linear estimation techniques.

Symbolically, each error e measured in a certain terminal slew attitude may be written as a linearized function of these unknown parameter changes $\Delta\xi$:

$$e = \sum_{\xi} C_{\xi} \Delta \xi$$

The coefficients C_{ξ}, which vary from observation to observation, may be collected into a row vector of coefficients, $\mathcal{C}_e^T$. The unknown changes $\Delta\xi$ in parameter values, which are assumed to remain the same from observation to observation, may be collected into a column vector of parameters, P:

$$P = \begin{bmatrix} \Delta d\omega_{21} \\ \Delta d\omega_{31} \\ \Delta d\omega_{12} \\ \Delta d\omega_{13} \\ \vdots \\ \Delta \zeta_{32} \\ \Delta \zeta_{13} \\ \Delta \zeta_{23} \end{bmatrix}$$

Then the error e may be expressed

$$e = [C_{d\omega_{21}} \quad C_{d\omega_{31}} \quad \cdots \quad C_{\zeta_{23}}] \begin{bmatrix} \Delta d\omega_{21} \\ \Delta d\omega_{31} \\ \vdots \\ \Delta \zeta_{23} \end{bmatrix}$$

If several errors e are available, from the same or other slews, one can write an equation like the above for each. The row of coefficients, $\mathcal{C}_e^T$, will vary as various errors are used, but the column of unknowns, P, is assumed to remain fixed. Now a large number of error observations from many different slews may be collected into a column array, E, and the rows $\mathcal{C}_e^T$ associated with

these errors can be assembled into a coefficient matrix $\mathcal{C}$. Then we can write the following, called the IRU Estimation Equation:

$$\mathcal{C}P = E$$

The above equation will be constructed operationally as follows. After the errors e_r, e_p, e_y in roll, pitch and yaw for a certain slew are read, the 36 coefficients $C_{r\xi}$, $C_{p\xi}$, $C_{y\xi}$ which relate these errors to the 12 unknown parameter changes $\Delta\xi$ are calculated. (Formulas for calculating these coefficients are derived later in this paper.) These data are then assembled into the next three rows of the Estimation Equation, as shown diagrammatically in Figure 7.

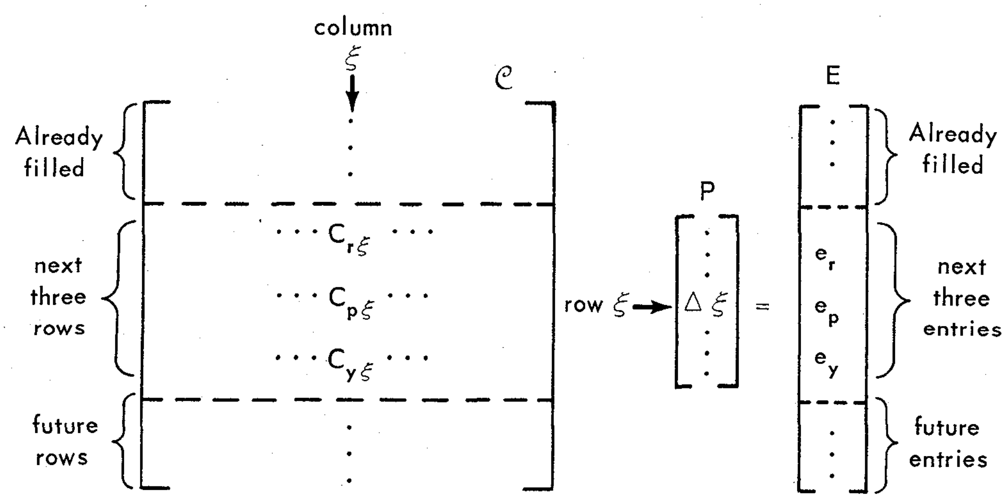


Figure 7. Data Placement in IRU Estimation Equation

The design philosophy of the IRU Exercise then is to provide a large number of slews of a wide range of angles about all three axes, with an error readout at the end of each slew. In this way sufficient error terms E can be collected and coefficients $\mathcal{C}$ computed for the Estimation Equation, to enable a valid statistical solution for the parameters P to be determined. The remainder of

this section will be devoted to developing formulas for computing the coefficients $\mathcal{C}$.

Suppose a slew is commanded, say a three-legged slew, from an initial stellar-referenced attitude, and the slew is allowed to take place under IRU control. Suppose that S_1, S_2, S_3 represent the nominal coordinate transforms resulting from the first, second, third legs, respectively. Suppose further that S_1', S_2', S_3' represent the actual coordinate transforms resulting from the mis-aligned slew legs. Then we have the coordinate system flow diagram shown in Figure 8.

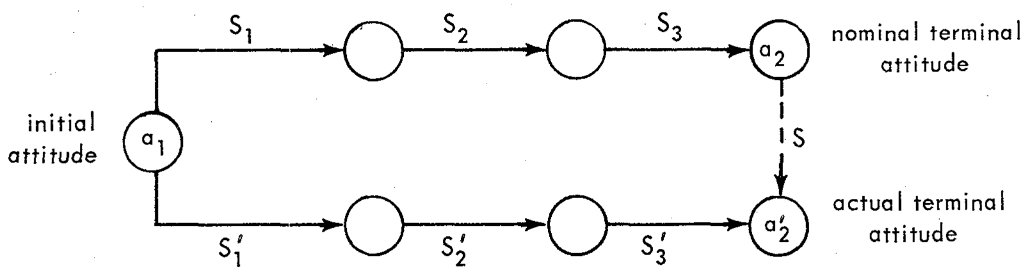


Figure 8. Coordinate Transformations Related to Slewing

In Figure 8, the transformation S from a_2 to a_2' is given by

$$S = S_3' \, S_2' \, S_1' \, S_1^T \, S_2^T \, S_3^T$$

The attitude a_2' is by definition the attitude in which the coordinate system which is stellar referenced at a_2 actually ended up. This coordinate reference was nominally to have ended up at a_2. Therefore if some combination of stellar-referenced sensors (GST, BST, FES) is allowed to control the spacecraft subsequent to settling out (at attitude a_2') from the IRU-controlled slew, the spacecraft will be brought (except for misalignment of the stellar reference) to

attitude a_2. When this happens, the IRU will take on roll, pitch and yaw errors e_r, e_p, e_y. These must therefore be the angles which determine the small-angle rotation S:

$$S = S_3' \, S_2' \, S_1' \, S_1^T \, S_2^T \, S_3^T = \begin{pmatrix} 1 & e_y & -e_p \\ -e_y & 1 & e_r \\ e_p & -e_r & 1 \end{pmatrix}$$

Each error e_r, e_p, e_y can be considered a function of the six nonorthogonality parameters $d\omega_{mn}$ (m, n = 1, 2, 3; m ≠ n) and the six slew calibration parameters ζ_{pq} (p = 1, 2, 3; q = 1, 2). We can write formally $e_s = e_s\,(d\omega_{12}, d\omega_{13}, \ldots, \zeta_{32})$, s = r, p, y. Since each e_s has nominal value $e_s = 0$, we can relate the errors e_s to changes $\Delta d\omega_{mn}$, $\Delta \zeta_{pq}$ in the parameters. To first order,

$$e_s = \left(\frac{\partial e_s}{\partial d\omega_{12}}\right)_o \Delta d\omega_{12} + \left(\frac{\partial e_s}{\partial d\omega_{13}}\right)_o \Delta d\omega_{13} + \cdots + \left(\frac{\partial e_s}{\partial \zeta_{32}}\right)_o \Delta \zeta_{32}$$

The above equation relates the changes $\Delta d\omega_{12}$, $\Delta d\omega_{13}$, . . ., $\Delta \zeta_{32}$ in the parameters of interest to errors e_r, e_p, e_y measured in the operating environment. The coefficients

$$\left(\frac{\partial e_s}{\partial d\omega_{12}}\right)_o, \left(\frac{\partial e_s}{\partial d\omega_{13}}\right)_o, \ldots, \left(\frac{\partial e_s}{\partial \zeta_{32}}\right)_o$$

are the desired coefficients $C_{s\xi}$ of the IRU Estimation Equation.

Let ξ be any of the 12 parameters to be estimated. Then we have

$$\frac{\partial S}{\partial \xi} = \frac{\partial}{\partial \xi} \begin{pmatrix} 1 & e_y & -e_p \\ -e_y & 1 & e_r \\ e_p & -e_r & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{\partial e_y}{\partial \xi} & -\frac{\partial e_p}{\partial \xi} \\ -\frac{\partial e_y}{\partial \xi} & 0 & \frac{\partial e_r}{\partial \xi} \\ \frac{\partial e_p}{\partial \xi} & -\frac{\partial e_r}{\partial \xi} & 0 \end{pmatrix}$$

$$= \frac{\partial}{\partial \xi} (S_3' \, S_2' \, S_1' \, S_1^T \, S_2^T \, S_3^T)$$

$$= \frac{\partial S_3'}{\partial \xi} \, S_2' \, S_1' \, S_1^T \, S_2^T \, S_3^T + S_3' \, \frac{\partial S_2'}{\partial \xi} \, S_1' \, S_1^T \, S_2^T \, S_3^T + S_3' \, S_2' \, \frac{\partial S_1'}{\partial \xi} \, S_1^T \, S_2^T \, S_3^T$$

Evaluating this equation at the nominal state 0, $S_i = S_i'$, $i = 1, 2, 3$, yields the desired coefficients

$$C_{s\xi} = \left(\frac{\partial e_s}{\partial \xi} \right)_0, \quad s = r, p, y$$

which appear as elements of the array

$$\begin{pmatrix} 0 & C_{y\xi} & -C_{p\xi} \\ -C_{y\xi} & 0 & C_{r\xi} \\ C_{p\xi} & -C_{r\xi} & 0 \end{pmatrix} = \left(\frac{\partial S_3'}{\partial \xi} \right)_0 S_3^T + S_3 \left(\frac{\partial S_2'}{\partial \xi} \right)_0 S_2^T \, S_3^T + S_3 \, S_2 \left(\frac{\partial S_1'}{\partial \xi} \right)_0 S_1^T \, S_2^T \, S_3^T$$

Hence the coefficients $C_{s\xi}$ depend on the slew S and the partials

$$\left(\frac{\partial S_i'}{\partial \xi}\right)_0, \quad i = 1, 2, 3$$

The subscripts i, 0 will be dropped hereinafter for readability.

If $\xi = d\omega_{mn}$ for some m, n, we will have $\partial S'/\partial\xi = 0$ unless the slew S' takes place about the n-axis. If the slew S' does take place about the n-axis, it is shown in the Appendix that

$$\frac{\partial S'}{\partial d\omega_{mn}} = (1 - C\theta)\,(\hat{e}_m \hat{e}_n^T + \hat{e}_n \hat{e}_m^T) - S\theta\, \tilde{\hat{e}}_m; \quad m, n = 1, 2, 3; \; m \neq n$$

where θ is the angle slewed, including algebraic sign; $\hat{e}_m$, $\hat{e}_n$ are the Kronecker basis vectors along the m, n axes, respectively; $\tilde{\hat{e}}_m$ is the slew-symmetric matrix associated with the vector $\hat{e}_m$ according to the correspondence

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longleftrightarrow \widetilde{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

If $\xi = \zeta_{pq}$ for some p, q, we will have $\partial S'/\partial\xi = 0$ unless the slew S' takes place about the p-axis in the q-direction. In this latter case, suppose a slew S of angle θ is commanded about the p-axis in the q-direction. Then the slew S' which will actually occur is $(1 + \zeta_{pq})\,\theta$, and it is shown in the Appendix that

$$\frac{\partial S'}{\partial \zeta_{pq}} = -\theta\,\left(C\theta\, \tilde{\hat{e}}_p + S\theta\,(I - \hat{e}_p \hat{e}_p^T)\right) \; ; \; p = 1, 2, 3; \; q = 1, 2$$

5. Acknowledgment

It is a pleasure to acknowledge the support and encouragement of Sal Soscia and Paul Davenport at Goddard Space Flight Center, and to recall the sometimes spirited conversations with the latter which ensured consistency in the modeling and accuracy in the analysis.

6. Appendix: Derivation of Formulas for Computing $\partial S'/\partial \xi$.

It is well known that the matrix S of a rotation about an axis represented by the unit vector

$$\hat{e} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

through the angle θ can be expressed in "axis-angle" form:

$$S = C\theta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (1 - C\theta) \begin{pmatrix} x_1^2 & x_1 x_2 & x_1 x_3 \\ x_1 x_2 & x_2^2 & x_2 x_3 \\ x_1 x_3 & x_2 x_3 & x_3^2 \end{pmatrix} + S\theta \begin{pmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{pmatrix}$$

$$= C\theta\, I + (1 - C\theta)\, \hat{e}\, \hat{e}^T - S\theta\, \tilde{\hat{e}}$$

where $\tilde{\hat{e}}$ is the skew-symmetric matrix associated with the vector $\hat{e}$ according to the correspondence

$$\hat{e} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \longleftrightarrow \tilde{\hat{e}} = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}.$$

To compute $\partial S'/\partial \xi$ when $\xi = d\omega_{ij}$, first observe that each misaligned coordinate vector $\hat{e}_n'$ is represented in the nominal coordinate system by the n^{th} column of the array $I + d\Omega$, where $d\Omega$ is the array of nonorthogonality parameters:

$$d\Omega = \begin{pmatrix} 0 & d\omega_{12} & d\omega_{13} \\ d\omega_{21} & 0 & d\omega_{23} \\ d\omega_{31} & d\omega_{32} & 0 \end{pmatrix}$$

Then a rotation about the misaligned axis e_n through angle θ has the matrix representation

$$S' = C\theta\, I + (1 - C\theta)\, \hat{e}_n'\, \hat{e}_n'^T - S\theta\, \tilde{\hat{e}}_n'$$

$$= C\theta\, I + (1 - C\theta)\, (I + d\Omega)_n\, (I + d\Omega)_n^T - S\theta\, (\widetilde{I} + \widetilde{d\Omega})_n$$

$$= C\theta\, I + (1 - C\theta)\, (I_n\, I_n^T + d\Omega_n\, I_n^T + I_n\, d\Omega_n^T) - S\theta\, (\tilde{I}_n + \widetilde{d\Omega}_n)$$

including terms through first order. Now only the parameters $d\omega_{mn}$, $m \neq n$, appear in $d\Omega_n$. For these parameters,

$$\frac{\partial\, d\Omega_n}{\partial\, d\omega_{mn}} = \tilde{\hat{e}}_m$$

and also we notice that I_n is just $\hat{e}_n$. Hence

$$\frac{\partial S'}{\partial\, d\omega_{mn}} = \frac{\partial}{\partial\, d\omega_{mn}} \left((d\Omega_n\, I_n^T + I_n\, d\Omega_n^T)\, (1 - C\theta) - S\theta\, \widetilde{d\Omega}_n \right)$$

$$= (\hat{e}_m\, \hat{e}_n^T + \hat{e}_n\, \hat{e}_m^T)\, (1 - C\theta) - S\theta\, \tilde{\hat{e}}_m$$

To compute $\partial S'/\partial \xi$ when $\xi = \zeta_{rq}$, again the axis-angle form of a slew is utilized. Now a slew S' which takes place about an axis represented by the unit vector $\hat{e}_p$ in the q-direction through an angle $(1 + \zeta_{pq})\,\theta$ has the form

$$S' = C\,(1 + \zeta_{pq})\,\theta\, I + (1 - \cos\,(1 + \zeta_{pq})\,\theta)\, \hat{e}_p\, \hat{e}_p^T - S\,(1 + \zeta_{pq})\,\theta\, \tilde{\hat{e}}_p$$

Differentiating yields

$$\frac{\partial S'}{\partial \zeta_{pq}} = -\theta\, S\,(1+\zeta_{pq})\,\theta\, I + \theta\, S\,(1+\zeta_{pq})\,\theta\, \hat{e}_p\, \hat{e}_p^T - \theta\, C\,(1+\zeta_{pq})\,\theta\, \tilde{\hat{e}}_p$$

$$= -\theta\left(C\,(1+\zeta_{pq})\,\theta\, \tilde{\hat{e}}_p + S\,(1+\zeta_{pq})\,\theta\,(I - \hat{e}_p\, \hat{e}_p^T)\right)$$

Since the derivative is needed only to zero order, this may be evaluated when

$\zeta_{pq} = 0$:

$$\left(\frac{\partial S'}{\partial \zeta_{pq}}\right)_o = -\theta\left(C\theta\, \tilde{\hat{e}}_p + S\theta\,(I - \hat{e}_p\, \hat{e}_p^T)\right).$$

7. References

Descriptions of the mathematical models of the OAO used in MESS have appeared in OAO-SCPS Technical Memorandum T-70-3, SCPS Misalignment Models for the OAO-B Attitude Control Sensors, by R. desJardins, March 23, 1970. Analysis for the subroutine PAKIRU has appeared in the OAO-SCPS Technical Memorandum T-70-4, Mathematical Analysis and Operational Considerations for the IRU Alignment Exercise, by R. desJardins, April 30, 1970. Analysis for the subroutine EQMISG, which performs the analytic computations for the calling subroutine PAKGST, has appeared in GSFC X-Document X-542-69-418, In-Orbit Startracker Misalignment Estimation on the OAO, by R. desJardins, September 1969.

The reader is referred to OAO-SCPS Technical Memorandum T-71-1, An Overview of MESS, by R. desJardins, January 11, 1971; OAO-SCPS Informal Memorandum I-71-11, MESS Operations, by R. desJardins and D. W. Greer, March 3, 1971; and OAO-SCPS Technical Memorandum T-71-3, Mathematical Analysis for MESS, by R. desJardins, June 29, 1971; for the original documents from which the bulk of the present paper was abstracted.